

Polarized and Unpolarized Structures of the Virtual Photon¹

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Abstract. We discuss the structure functions and the parton distributions in the virtual photon target, both polarized and unpolarized, beyond the leading order in QCD. We study the factorization-scheme dependence of the parton distributions.

INTRODUCTION

As Maria Krawczyk remarked in her introductory talk on structure functions [1], the virtual photon structure provides a unique test of QCD. In this talk I would like to discuss the polarized and unpolarized virtual photon structures. But because of the limitation of the allocated time, I will mainly focus my talk on the polarized virtual photon structure.

Recently there has been growing interest in the polarized photon structure functions. Especially, the 1st moment of a photon structure function g_1^γ has attracted much attention in connection with its relevance for the axial anomaly, which has also played an important role in the QCD analysis of the nucleon spin structure functions. Now the information on the spin structure of the photon will be obtained from the resolved photon process in polarized electron and proton collision in the polarized version of the ep collider. More directly, the spin-dependent structure function of the photon can be measured by the polarized e^+e^- collision in the future linear colliders.

Here we investigate two-photon process (Figure 1) with the kinematical region where the mass squared of the probe photon (Q^2) is much larger than that of the target photon (P^2) which is in turn much bigger than the Λ^2 , the QCD scale parameter squared. The advantage for studying the virtual photon target is that we can calculate whole structure functions up to next-leading-order (NLO), in contrast to the real photon target where there remain uncalculable non-perturbative pieces. This is true for summing up the QCD logarithmic terms due to twist-2 operators corresponding to the QCD parton picture. Here we neglect all the power corrections arising from the higher-twist effects and target mass effects of the form $(P^2/Q^2)^k$

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($k = 1, 2, \dots$). Some non-perturbative effects like gluon condensations reside in the higher-twist effects. Our aim here is to study the polarized virtual photon structure function $g_1^\gamma(x, Q^2, P^2)$ at the same level of unpolarized structure function $F_2^\gamma(x, Q^2, P^2)$ (For the experimental status, see [2]). We can also investigate the parton distributions inside the polarized virtual photon. As we will see, the spin structure of the polarized virtual photon would offer a good testing ground for factorization scheme dependence of the parton distribution functions.

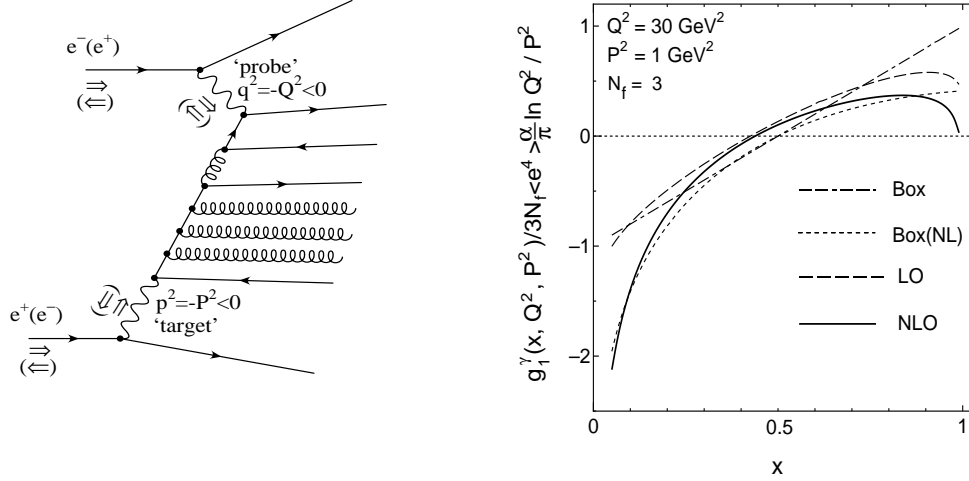


FIGURE 1. Two-photon process in polarized e^+e^- collision for $\Lambda^2 \ll P^2 \ll Q^2$ and the polarized photon structure function $g_1^\gamma(x, Q^2, P^2)$ for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ with $N_f = 3$. LO, NLO and Box (NL) denote QCD LO, NLO and Box-diagram (non-leading) results.

PQCD CALCULATION

We can apply the same framework used in the analysis of nucleon spin structure functions, namely the operator product expansion (OPE) supplemented by the renormalization group (RG) method or equivalently DGLAP type parton evolution equations. The NLO calculation has become possible since the two-loop anomalous dimensions of the quark and gluon operators in OPE or equivalently two-loop parton splitting functions were calculated by two groups [3,4]. The n -th moment of $g_1^\gamma(x, Q^2, P^2)$ for the kinematical region:

$$\Lambda^2 \ll P^2 \ll Q^2$$

is given by

$$\begin{aligned} \int_0^1 dx x^{n-1} g_1^\gamma(x, Q^2, P^2) = & \frac{\alpha}{4\pi} \frac{1}{2\beta_0} \left[\sum_{i=+, -, NS} L_i^n \frac{4\pi}{\alpha_s(Q^2)} \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n / 2\beta_0 + 1} \right\} \right. \\ & + \sum_{i=+, -, NS} \mathcal{A}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n / 2\beta_0} \right\} + \sum_{i=+, -, NS} \mathcal{B}_i^n \left\{ 1 - \left(\frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\lambda_i^n / 2\beta_0 + 1} \right\} \\ & \left. + \mathcal{C}^n + \mathcal{O}(\alpha_s) \right] \end{aligned} \quad (1)$$

where L_i^n , \mathcal{A}_i^n , \mathcal{B}_i^n and \mathcal{C}^n are computed from the 1- and 2-loop anomalous dimensions as well as from 1-loop coefficient functions. λ_i^n ($i = +, -, NS$) denote the eigenvalues of 1-loop anomalous dimension matrices. $\alpha_s(Q^2)$ is the QCD running coupling constant. In Figure 1 we have shown the $g_1^\gamma(x, Q^2, P^2)$ evaluated from (1) by inverse Mellin transform for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ with $N_f = 3$ [10]. Note that the same formula with different coefficients, L_i^n , \mathcal{A}_i^n , \mathcal{B}_i^n , \mathcal{C}^n and λ_i^n ($i = +, -, NS$) holds for the unpolarized structure function $F_2^\gamma(x, Q^2, P^2)$ [5].

SUM RULE

For a real photon target ($P^2 = 0$), Bass, Brodsky and Schmidt have shown that the 1st moment of $g_1^\gamma(x, Q^2)$ vanishes to all orders of $\alpha_s(Q^2)$ in QCD [6]:

$$\int_0^1 dx g_1^\gamma(x, Q^2) = 0. \quad (2)$$

Now the question is what about the $n = 1$ moment of the virtual photon case. Here we note that the eigenvalues of one-loop anomalous dimension matrix are $\lambda_+^{n=1} = 0$, $\lambda_-^{n=1} = -2\beta_0$. Taking $n \rightarrow 1$ limit of (1) the first three terms vanish. Denoting e_i , the i -th quark charge and N_f , the number of active flavors, we have

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{N_f} e_i^4 + \mathcal{O}(\alpha_s) \quad (3)$$

We can go a step further to $\mathcal{O}(\alpha_s)$ QCD corrections which turn out to be [7]:

$$\int_0^1 dx g_1^\gamma(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \left[\sum_{i=1}^{N_f} e_i^4 \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) - \frac{2}{\beta_0} \left(\sum_{i=1}^{N_f} e_i^2 \right)^2 \left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi} \right) \right] + \mathcal{O}(\alpha_s^2). \quad (4)$$

This result coincides with the one obtained by Narison, Shore and Veneziano [8], apart from the overall sign for the definition of g_1^γ .

PARTON DISTRIBUTIONS

Spin-dependent parton distributions

Factorization theorem tells us that the physically observable quantities like cross sections or structure functions can be factored into the long-distance part (distribution function) and short-distance part (coefficient function). Thus the polarized photon structure function can be written schematically as

$$g_1^\gamma = \Delta \vec{q}^\gamma \otimes \Delta \vec{C}^\gamma \quad (5)$$

where spin-dependent parton distributions $\Delta\vec{q}$:

$$\Delta\vec{q}^\gamma(x, Q^2, P^2) = (\Delta q_S^\gamma, \Delta G^\gamma, \Delta q_{NS}^\gamma, \Delta\Gamma^\gamma) \quad (6)$$

are polarized flavor-singlet quark, gluon, non-singlet quark and photon distribution functions in the virtual photon (we put the symbol Δ for polarized quantities), and

$$\Delta\vec{C}^\gamma = \begin{pmatrix} \Delta C_S^\gamma \\ \Delta C_G^\gamma \\ \Delta C_{NS}^\gamma \\ \Delta C_\gamma^\gamma \end{pmatrix} \quad (7)$$

are the corresponding coefficient functions. The same relation holds for unpolarized structure function F_2^γ in terms of unpolarized parton distributions \vec{q} and unpolarized coefficient functions \vec{C}^γ . In the leading order in QED coupling $\alpha = \frac{e^2}{4\pi}$, the photon distribution function can be taken as $\Delta\Gamma^\gamma(x, Q^2, P^2) = \delta(1-x)$. Therefore we have the following inhomogeneous DGLAP evolution equation for $\Delta\vec{q}^\gamma = (\Delta q_S^\gamma, \Delta G^\gamma, \Delta q_{NS}^\gamma)$:

$$\frac{d\Delta\vec{q}^\gamma(x, Q^2, P^2)}{d\ln Q^2} = \Delta\mathbf{K}(x, Q^2) + \int_x^1 \frac{dy}{y} \Delta\vec{q}^\gamma(y, Q^2, P^2) \times \Delta P\left(\frac{x}{y}, Q^2\right) \quad (8)$$

where $\Delta\mathbf{K}(x, Q^2)$ is the splitting function of the photon into quark and gluon, whereas $\Delta P(x/y, Q^2)$ is the 3×3 splitting function matrix.

Factorization Scheme Dependence

The solution to the DGLAP evolution equation can be given by

$$\Delta\vec{q}^\gamma(t) = \Delta\vec{q}^{\gamma(0)}(t) + \Delta\vec{q}^{\gamma(1)}(t), \quad t \equiv \frac{2}{\beta_0} \ln \frac{\alpha_s(P^2)}{\alpha_s(Q^2)} \quad (9)$$

where the first (second) term corresponds to LO (NLO) approximation. The initial condition we impose is the following,

$$\Delta\vec{q}^{\gamma(0)}(0) = 0, \quad \Delta\vec{q}^{\gamma(1)}(0) = \frac{\alpha}{4\pi} \vec{A}_n \quad (10)$$

where \vec{A}_n is the constant which depends on the factorization scheme to be used. Or equivalently in the language of OPE, this constant appears as a finite matrix element of the operators, \vec{O}_n renormalized at $\mu^2 = P^2$ between the photon states:

$$\langle \gamma(p) | \vec{O}_n(\mu) | \gamma(p) \rangle|_{\mu^2=P^2} = \frac{\alpha}{4\pi} \vec{A}_n \quad (11)$$

This scheme dependence arises from the freedom of multiplying the arbitrary finite renormalization constant Z_a and its inverse Z_a^{-1} in the n -th moment of (5):

$$g_1^\gamma(n, Q^2, P^2) = \Delta\vec{q}^\gamma \cdot \Delta\vec{C}^\gamma = \Delta\vec{q}^\gamma Z_a \cdot Z_a^{-1} \Delta\vec{C}^\gamma = \Delta\vec{q}^\gamma|_a \cdot \Delta\vec{C}^\gamma|_a \quad (12)$$

where the resulting $\Delta\vec{q}^\gamma|_a$ and $\Delta\vec{C}^\gamma|_a$ are the distribution function and the coefficient function in the a -scheme. The explicit expressions for the n -th moment of the parton distributions can be found in ref. [10].

Transformation from $\overline{\text{MS}}$ to a -scheme

Under the transformation from one factorization scheme to another, the coefficient functions as well as anomalous dimensions will change. Of course when they are combined together, we get the factorization-scheme independent structure function g_1^γ . Since $\overline{\text{MS}}$ is the only scheme in which both 1-loop coefficient functions and 2-loop anomalous dimensions are actually computed, we study the transformation rule from the $\overline{\text{MS}}$ to a new factorization scheme- a . We have considered the several different factorization schemes; 1) chirally invariant (CI) scheme, 2) Adler-Bardeen (AB) scheme, 3) off-shell (OS) scheme, 4) Altarelli-Ross (AR) scheme and 5) DIS_γ scheme. (For the detailed description of each factorization scheme see [10].) The transformation rule for the singlet-quark coefficient function, for example, is given by

$$\Delta C_{S,a}^{\gamma,n} = \Delta C_{S,\overline{\text{MS}}}^{\gamma,n} - \langle e^2 \rangle \frac{\alpha_s}{2\pi} \Delta w(n,a) \quad (13)$$

where $\langle e^2 \rangle = \sum_i e_i^2/N_f$ and $\Delta w(n,a)$ is the transformation functions, the explicit expressions for which as well as other coefficient functions together with the similar transformation rules for 2-loop anomalous dimensions are given in ref. [10].

The prescriptions to treat the axial anomaly are different from scheme to scheme. For example, the axial anomaly resides in the quark distribution in $\overline{\text{MS}}$ scheme, whereas it exists in the gluon and photon coefficient functions in the CI scheme. These factorization schemes are also characterized by the behavior of the parton distribution functions near $x = 1$. We can study their analytic behaviors by the large n limit of their moments. Here we also note that the gluon distribution function is factorization-scheme independent in the class of factorization schemes considered here. By performing the inverse Mellin transform of the moments, the parton distributions as functions of x are reproduced numerically. We present our results for singlet-quark for various schemes and gluon in Figure 2. Note that the real photon's g_1^γ was studied in [11,12], which are consistent with present analysis.

The similar scheme dependence of the parton distributions inside the unpolarized virtual photon was studied for the $\overline{\text{MS}}$, OS and DIS_γ schemes [13].

CONCLUDING REMARKS

We have studied the virtual photon's spin structure functions, $g_1^\gamma(x, Q^2, P^2)$ and the polarized parton distributions for the kinematical region $\Lambda^2 \ll P^2 \ll Q^2$, which are perturbatively calculable up to the NLO in QCD. The first moment of g_1^γ is non-vanishing in contrast to the real photon case, where we have vanishing sum rule. NLO QCD corrections are significant at large x as well as at low x . We also studied factorization-scheme dependence of parton distribution functions.

Future subjects to be studied are as follows. First of all we should understand how the transition occurs from vanishing 1st moment for real photon ($P^2 = 0$) to

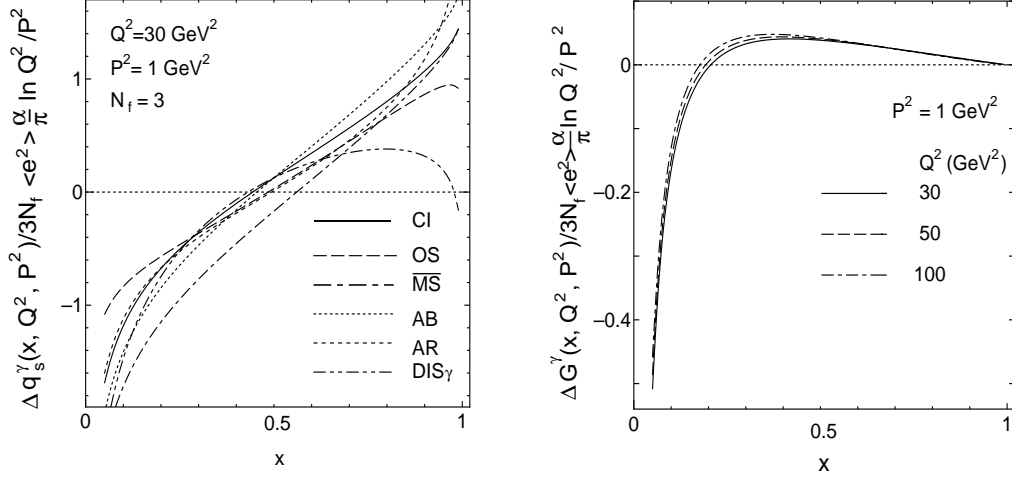


FIGURE 2. Polarized singlet-quark distribution function $\Delta q_s^\gamma(x, Q^2, P^2)$ in the several factorization schemes and the polarized gluon distribution $\Delta G^\gamma(x, Q^2, P^2)$ for $Q^2 = 30 \text{ GeV}^2$ and $P^2 = 1 \text{ GeV}^2$ with $N_f = 3$

non-vanishing one for virtual photon ($P^2 \gg \Lambda^2$). Secondly, another structure function $g_2^\gamma(x, Q^2, P^2)$ is yet to be computed where we also have twist-3 contribution. Furthermore, the power corrections due to target mass effects and higher-twist effects should be investigated. More reliable treatment for small- x behaviors of polarized p.d.f. should be studied in the framework of BFKL like approach.

REFERENCES

1. Krawczyk, M., these proceedings.
2. Söldner-Rembold, S., these proceedings.
3. Mertig, R., and van Neerven, W. L., *Z. Phys.* **C70**, 637 (1996).
4. Vogelsang, W., *Phys. Rev.* **D54**, 2023 (1996).
5. Uematsu, T., and Walsh, T. F., *Nucl. Phys.* **B199**, 93 (1982).
6. Bass, S. D., Brodsky, S. J., and Schmidt, I., *Phys. Lett.* **B437**, 417 (1998).
7. Sasaki, K., and Uematsu, T., *Phys. Rev.* **D59**, 114011 (1999).
8. Narison, S., Shore, G. M., and Veneziano, G., *Nucl. Phys.* **B391**, 69 (1993).
9. Sasaki, K., and Uematsu, T., *Phys. Lett.* **B473**, 309 (2000).
10. Sasaki, K., and Uematsu, T., hep-ph/0007055 (2000).
11. Sasaki, K., *Phys. Rev.* **D22**, 2143 (1980); *Prog. Theor. Phys. Suppl.* **77**, 197 (1983).
12. Stratmann, M., and Vogelsang, W., *Phys. Lett.* **B386**, 370 (1996).
13. Sasaki, K., and Uematsu, T., *Nucl. Phys. B (Proc. Suppl.)* **89**, 162 (2000).